LEARNING FROM THEIR MISTAKES (HSC 2001): ADVICE FOR STUDENTS

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Each year, the Board of Studies publishes an examination report describing the performance of students in the previous year's HSC mathematics exams, outlining their strengths and weaknesses, common errors and deficiencies, the marking scheme for every question, and advice to this year's HSC students and teachers. The exams and reports for the 2001 HSC (and other past HSCs) can be downloaded from the Board of Studies website at *www.boardofstudies.nsw.edu.au/hsc_exams*. Also, the Mathematical Association of NSW (MANSW) publishes inexpensive booklets of past HSC exams with worked solutions (phone 9878-1487 or website *www.hsc.csu.edu.au/pta/mansw*): you may be able to purchase these through your school's mathematics faculty.

On the first Saturday of March each year, MANSW also conducts an HSC examiners' day at Macquarie University (open to all), where senior markers report on and analyse students' responses on the previous year's HSC exam. This article extracts the main themes from the 2001 HSC exam reports and examiners day, providing HSC tips for this year's students and teachers. It has been rewritten annually since 1997 and is based on my 1995 *Reflections* article, "Ten common mistakes made by HSC Maths students."

Ten tips for this year's HSC mathematics students

1. Show your working

The most common mistake made by HSC students is to write an answer with little or no working. By now, your teacher will have reminded you that a wrong answer with no working (what HSC markers call a "bald" answer) receives no marks while a wrong answer with correct working may earn some marks. The examiner marking your paper looks for evidence that you know what you're doing, trying to grant you as many marks as possible. He uses a marking scheme that shows the correct solution to the question and where each progressive mark for that question is awarded. For example, Question 1(a) of the (2 unit) Mathematics exam was worth 2 marks:

Evaluate, correct to three significant figures, $\sqrt{\dfrac{3^2+12^2}{231-12^2}}$.

The first mark was given for correctly evaluating the expression, the second mark for correct rounding to 3 significant figures. So the correct answer of 1.33 earned full marks while an incomplete answer ($\sqrt{\dfrac{153}{87}}$) or incorrectly-rounded answer (1.326) received one mark. Even an incorrect answer such as 0.142 was granted one mark because it was correctly rounded to 3 significant figures.

This example illustrates the importance of showing all working rather than just writing a bald answer. Each progressive mark for a question is awarded when you have pass each stage in its solution, so it pays to set out your work as neatly as possible, with each step listed clearly and spread out on the page. Also, you should not scribble over or erase incorrect or rough working. Instead, a neat line crossed through it might help examiners understand your thinking because sometimes, even crossed-out work can earn marks.

2. Pay attention to the number of marks allocated to a question

So how much working is necessary? The number of marks allocated to a question is a good guide to the number of things you need to show and the level of difficulty of the question. Most questions are worth 2 marks—one mark for correct working and another mark for the correct answer. One-mark questions are usually simple ones that require you to state a fact, make a quick calculation or write a one-line answer. Three examples of 1-mark questions from last year's exams:

Question 24(a)(i), General Mathematics (GM): What is the size of ∠AOB?
Question 25(b)(iv) 2, GM: Describe a method that could have been used to select a random sample of the male students.
Question 2(b)(ii), Mathematics (M): Show that the length of AB is $2\sqrt{10}$.

Beware of writing long answers to 1- or 2-mark questions, especially when the question asks you to justify or explain something in words rather than numbers. In the General Mathematics questions:

Question 25(a)(iii): Antoinette ... concludes she has a good chance of remaining. Do you agree? Justify your answer. [2 marks]
Question 23(c)(iii): Compare and contrast the two data sets by examining the shape and skewness of the distributions, and the measures of location and spread, [3 marks]

some students wrote 2-page essays. The examiners commented that those students who answered the question succinctly in dot point or table form were generally the most successful.

3. Don't neglect the Preliminary course

The Preliminary and HSC courses in mathematics are inter-related more than in any other subject, with the HSC topics building upon knowledge learned in the Preliminary topics. Furthermore, up to 30% of the General Mathematics exam and up to 20% of the Mathematics exam can test knowledge and skills from the Preliminary course, so do not forget to study the Preliminary course.

In last year's General Mathematics exam, there were many questions testing the understanding of Preliminary work in Algebra, Measurement and Data Analysis. For example, many students were caught out when they did not revise their understanding of the cumulative frequency histogram (Question 25(b)) from the Preliminary course.

Twelve marks (out of 120 marks) of last year's Mathematics exam tested Basic Arithmetic and Algebra (including surds) from the Preliminary course, with Linear Functions (coordinate geometry) and Plane Geometry also featuring prominently. Even The Quadratic Polynomial and the Parabola made up 6 marks of the exam, but examiners reported that there were many students who made no attempt at these questions.

4. Don't forget your basic Years 9-10 skills, especially in algebra

Every year, the examiners are dismayed by the amount of simple algebraic and calculation errors made by HSC students, in:

- simplifying algebraic expressions
- substituting into expressions and formulas
- solving all types of equations
- factorising simple and quadratic expressions (Mathematics students only)
- surds (Mathematics students only)
- adding and multiplying fractions in probability problems
- graphing lines on a number plane

so you may need to include the practice of some basic mathematical skills in your study program. When substituting into an expression or formula, the examiners that you write the substitution line, listing the values being substituted. Some General Mathematics students last year also did not know that the symbol $\doteqdot$ meant "approximately equal to" and not "divided by" (÷) (Question 27(d)).

5. Learn to use your calculator properly

Make sure you know how to use your calculator competently in evaluating long numerical expressions that involve order of operations, such as $42000(1 - 0.15)^4$ and $\$680 \times \left\{ \frac{(1.0052)^{120} - 1}{0.0052 \times (1.0052)^{120}} \right\}$ in General Mathematics (Questions 27(b) and (d) and $\sqrt{\frac{3^2 + 12^2}{231 - 12^2}}$ and $70^{\frac{2}{3}}$ in Mathematics (Questions 1(a) and 3(b)).

Students are also reminded of the dangers of rounding off too early in the middle of a calculation when it should be done only *once* and at the very end. When calculating partial answers such as the numerators or denominators of the above expressions, do not round them off otherwise your final answer will be inaccurate. Keep partial answers in your calculator's display or memory, or use the ANS key if it has one, or write them down on paper with *all* decimal places so that you can recall them in further calculations. In last year's Mathematics exam, round-off errors occurred in final results when students used a rounded value of k in the exponential growth Question 8(a) or rounded partial answers in the superannuation calculations of Question 10(a).

For trigonometric calculations, make sure that your calculator are set in the correct mode: degrees or radians. Most calculators use degrees as the default mode, but some graphics calculators have radians as the default. General Mathematics students always use degrees, while Mathematics students use radians for arc length (Question 5(c)), the area of a sector and the calculus of trigonometric functions (Question 4(c)(iii) and Mathematics Extension 1, Question 3(a)).

A note to General Mathematics students about graphics calculators. Your HSC exam is designed to be "graphics calculator neutral," meaning that those students who use a scientific calculator in the exam will not be unfairly disadvantaged over those who use graphics calculators. The examiners commented that this was true in 2001, with the first General Mathematics HSC exam.

6. Read and answer the whole question

Sometimes, students don't read an HSC question completely and go ahead and answer what they think the question is asking or leave out some important part of the answer. The GST question in the General Mathematics paper (Question 27(a)) asked:

> George buys a television for $574.20, including 10% GST. What is the value of the GST component?

According to the exam report, the vast majority of students answered this part incorrectly. Most misread the question, thinking that $574.20 was the price *before* GST and calculated 10% of $574.20. Some other students correctly understood that the $574.20 *included* the GST, but then gave the price of the TV set before GST rather than the GST amount itself. As this was a one-mark question, only the correct answer of $52.20 was accepted.

In Question 1(a) of the Mathematics paper, students had to evaluate $\sqrt{\frac{3^2 + 12^2}{231 - 12^2}}$ correct to three significant figures, but some students rounded to three decimal places instead.

Students should be aware of double-barrelled HSC questions, where two questions are asked in one. Quite a number of students were caught out by the following questions from the Mathematics exam, answering one part but neglecting to answer the other:

> Question 3(d): Use the cosine rule to show that $x^2 - 7x = 120$, and hence find the exact value of *x*. [4 marks]
>
> Question 6(c)(ii): For what values of x is the curve concave up? Give reasons for your answer. [2 marks]

Question 8(c)(i): Find the appropriate depth y_1 at which $\frac{dy}{dt}$ is a maximum.

Find the appropriate depth y_2 at which $\frac{dy}{dt}$ is a minimum. [2 marks]

In Question 6(c)(ii), only 2% of students scored full marks. Those who found the point of inflexion but did not test for change in concavity received no marks.

When reading an exam question, identify exactly what needs to be done, using a highlighter to mark key words if needed. Then after writing the solution, double-check that you have actually answered the question.

7. Remember that the parts of a question are related

Most HSC questions have related parts, that is, the answer to part (i) leads to the solution of (ii) and so on. This is often the case with the coordinate geometry question in the Mathematics paper (usually Question 2), and last year's was no exception:

Question 2(b):
(i) Show that the equation of AB is $x + 3y - 13 = 0$.
(ii) Show the length of AB is $2\sqrt{10}$.
(iii) Calculate the perpendicular distance from O to the line AB
(iv) Calculate the area of parallelogram OABC

where the answer to (iii) required the answer to (i) and the answer to (iv) required the answers to (ii) and (iii). This was also true for Question 5(b) involving logarithms, Question 5(d) involving rates and volume, and Question 9(a) involving geometry and trigonometry. In the General Maths exam, it usually applies to the sine or cosine rule question:

Question 24(a):
(i) What is the size of $\angle AOB$?
(iii) Find the distance from A to B.

with (iii) requiring the use of the cosine rule with the angle in (i).

In these kind of questions, you are being lead step-by-step through the solution, with hints kindly provided the way. It would seem quite odd otherwise to be asked to find three or four unrelated things in the same question. Students who fail to notice that the parts of a question are related often use more complicated or impractical methods of solution that are longer, more time-consuming and prone to error. Some even end up "re-inventing the wheel," repeating their working from earlier parts.

8. Bring a ruler to the exam!

Each year, examiners are amazed by the number of students who fail to bring essential equipment such as a ruler to the HSC exam, resulting in poorly-drawn graphs and diagrams. General Mathematics students need a ruler to construct or read statistical and number plane graphs and to measure lengths and distances on scale diagrams (Question 24(c)), while Mathematics students need one to draw number plane graphs and geometrical diagrams in proofs.

Question 23 (a)(ii), General Mathematics: Draw a line of fit on your scatterplot on the graph paper provided.

Question 26 (a) (iii), General Mathematics: Draw a graph of the function $s = 220 - 4d$. Use your ruler to draw the axes. Label each axis, and mark a scale on each axis.

Question 4(b), Mathematics: Copy or trace the diagram. [Geometry proof]

Question 4(c) (i), Mathematics: Sketch the curve $y = 3 \sin 2x$ for $-\pi/2 \leq x \leq \pi/2$.

Across all exams there was evidence of graphs that were too small or messy to read, drawn freehand without much effort or attention to detail, without proper scales or labels on axes (despite the instructions given in Question 26 (a) (iii) above). Students need to bring the necessary equipment to draw big, clear diagrams. In Question 4(c) above, examiners reported that (p.8) "the best sketches of $y = 3 \sin 2x$ were achieved by those candidates who used a template, and then fitted the appropriate scales."

9. Learn compass bearings and the sine and cosine rules

One topic that features prominently in both the General Mathematics and the Mathematics HSC exams is trigonometry. The sine or cosine rule is tested almost every year, usually in a problem involving bearings. These are areas in which many students experience difficulty, so you should make an effort to revise them in your study.

In General Mathematics, Question 24(a) involved a radial survey in which an angle had to be found, the area of a triangle calculated using the $A = \frac{1}{2}ab \sin C$ formula, and the distance between two points found using the cosine rule. Common errors included using the sine rule or Pythagoras' theorem instead of the cosine rule, substituting wrong angle values, failure to relate the parts of the question, calculation mistakes and limited understanding of bearings. The sine rule was tested Question 20.

In the Mathematics exam, the cosine rule was actually applied twice, in Questions 3(d) and 9(a). The common error for Question 3(d) was in substituting inappropriate side values for the cosine rule.

10. Learn to draw and use a probability tree diagram

Probability is the other topic that is tested frequently in both General Mathematics and Mathematics. Most probability problems can be solved with a tree diagram even if it is not mentioned explicitly in the question, so again, a good study program should include practice in drawing and using tree diagrams.

Question 25 (a) (ii) 1, General Mathematics: Copy the tree diagram, and complete the diagram by writing the probabilities on all the branches.

Question 7 (b) (i), Mathematics: What is the probability that he connects for the first time on his second attempt?

In Question 25 above, examiners reported that many students read "complete the diagram" to mean add more branches to it rather than write the probabilities on the branches. Students also did not take into account that after the first selection there would be one less person, and wrote incorrect fractions on the second level of branches.

Some students still have trouble knowing how to use a tree diagram to solve a problem—what to do with the probabilities on the branches. Common mistakes include adding them instead of multiplying or vice-versa, or adding or multiplying fractions incorrectly without the aid of a calculator. The correct answer to Question 7(b) of the Mathematics paper was 0.25 x 0.25 x 0.25 = 0.015625, but many students rounded this answer unnecessarily to 0.016, scoring zero marks for an incorrect bald answer as this was a one-mark question.

Six more tips for General Mathematics students

1. Learn the language of the General Maths topics

General Mathematics exams are often wordy, with a lot of reading involved in the questions. This is because the mathematics is being applied to a variety of areas, each with its specific

terminology and jargon. Here are some phrases you may find in your HSC exam (do you know the meanings of each one?):

Financial mathematics: compounded monthly, ordinary annuity, contribution, present value, salvage value, depreciation.

Data analysis: measures of location and spread, five-number summary, consistent, clustering, skewness, outlier, quartile, median regression line.

Measurement: angle of depression, bearing, offset survey, two applications of Simpson's rule, great circle distance, "ignore time zones".

Probability: at least two, more than two, without replacement, ordered selection, financial expectation.

Algebraic modelling: change the subject, varies inversely, vertical intercept, line of best fit.

Question 23 (c) (iii) of last year's HSC paper asked students to compare the measures of location and spread of the heights of trees in two different parks. Some students did not know that "measures of location" referred to the statistical averages such as mean, median and mode while "measures of spread" referred to range, interquartile range or standard deviation. Some even mistook "measures of location" to mean the *physical* locations of the two parks: Central Park and East Park.

In Question 23 (a), students were asked to draw (i) a scatterplot, (ii) a line of fit on the scatterplot and (iii) to describe the correlation shown by the scatterplot. There were students who did not know what a scatterplot was (drawing a dot plot or column graph instead), or how to draw a line of fit (they simply joined the dots), or that the answer to (iii) was simply "a negative correlation," instead writing mini-essays for the one-mark question. Some students constructed a median regression line rather than a simple line of fit, a time-consuming process for another one-mark question. Some had difficulty using statistical jargon, confusing the term "spread" with "skewness" or "outliers", for example.

Not all misunderstandings came from specific jargon. Some came from plain English. In Question 25 (a) (ii) 2:

> Calculate the probability that the selection includes exactly one female.

some students misinterpreted "exactly one female" to include "two females." In probability problems, it is very important that you understand the subtle differences between phrases such as "at least one," "at most one," "more than one" and "exactly one".

As well as mastering your mathematical skills, you should spend considerable study time learning the language of General Mathematics. A personally-compiled glossary of key terms would be a valuable addition to your course notes.

2. Be prepared to explain your answers in words

Many General Mathematics questions require an interpretative worded answer rather than a calculated numerical one, so you must practise your writing skills. There were many explain/describe/justify/compare questions in last year's exam:

Question 23 (a) (iii): Describe the correlation [1 mark]

Question 23 (b) (i): Interpret this z-score in terms of the mean and standard deviation of the test. [2 marks]

Question 23 (c) (iii): Compare and contrast the two data sets [3 marks]

Question 25 (a) (iii): Do you agree? Justify your answer. [2 marks]

Question 25 (b) (iv) 1: Explain why this would NOT be a random sample. [1 mark] 2: Describe a method that could have been used to select a random sample of the male students. [1 mark]

Question 26 (a) (iv): Does it make sense to use the formula $s = 220 - 4d$ to calculate the number of stalls rented if Otto charges \$60 per stall? Explain your answer. [1 mark]

Question 27 (c) (i): Explain the large difference between the values of their investments [annuities problem] [1 mark]

Question 27 (d) (i): Explain why 0.0052 is used to calculate the monthly interest. [1 mark]

From the exam report (pp.5-6):

> Candidates generally have difficulty writing about their mathematical thinking clearly and concisely (and need to) become familiar with words such as 'explain', 'justify', 'compare' and 'contrast' as presented in the Board of Studies glossary of key words. Candidates also need to pay attention to the number of marks given to each part of a question ... A simple yes/no is not enough for a three-mark question, whereas a three-page explanation for one mark is not necessary.

Question 23(a) described the negative correlation between batting order and runs scored by each batter on a cricket team, but many "cricket commentators" went too far and wrote essays on their knowledge of the game and the current progress of the Australian cricket team, which was irrelevant to the question. Question 25(a) asked for the probability of a woman being randomly selected from a group of 8 people to leave an island, but too many "TV reviewers" decided to give their own opinions about the American TV series *Survivor*. Again, too much (unnecessary) information! Question 23(c) analysed the heights of trees in two parks, but instead of examining the given data, some students submitted 2-page stories about the skills of the council workers, the dates of tree-planting and the amount of rain and sunshine in both parks. All this for a 3-mark question!

In these situations, stay focused on answering the question and be careful you don't waffle on. As mentioned previously, presenting your arguments concisely in dot point or table form can earn good marks. Stick with the facts—your job is to analyse and interpret the calculated results, not to give some personal opinion with no mathematical evidence. Let's make one point perfectly clear: there is no place for creative writing in a mathematics exam!

3. Know your measurement and trigonometry

General Mathematics students should brush up on their Measurement knowledge and skills, especially in trigonometry and spherical (Earth) geometry. Question 24 of last year's paper involved (a) a radial survey (5 marks), (b) latitude and longitude on the Earth (4 marks) and (c) a scale drawing of a block of land (3 marks). Some students had difficulty applying the $A = \frac{1}{2}ab \sin C$ and cosine rule formulas in part (a), while part (b) was poorly done with many non-attempts, indicating that many students did not have a clear understanding of spherical geometry concepts such as time difference, latitude and great circle distance. In part (c), students neglected to use the scale ratio 1 : 250 or had difficulty converting between centimetres and metres, or between square centimetres and square metres. In the latter case, it is simpler and less error-prone to convert all lengths to metres first, then calculate area rather than calculate area in cm^2 and then convert to m^2.

4. Know how to read (and construct) the scale on a graph

Many students have difficulty interpreting the scale on the axes of a graph (number plane or statistical) and made mistakes in Question 26(b) which featured a graph of inflation over time. The price of a surfboard was shown on the vertical axis in intervals of 200 while the time was shown on the horizontal axis in intervals of 2 years. Students had trouble reading "in-between" values on both axes.

When students graphed the line $s = 220 - 4d$ in Question 26 (a) (iii), the examiners noted that (p.10):

> Scales (often freehand) and labelling were often incomplete or inaccurately done. A common error on the *d* axis was marking 0, 10, 30 and 50 at even intervals.

5. Know your algebra

Last year, 25 marks (out of 100) of the General Mathematics HSC exam dealt with Algebraic Modelling topics. Eleven marks tested Modelling Linear and Non-linear Relationships, so make sure you study this topic well. You should be familiar with the names, graphs and properties of the linear, quadratic, cubic, exponential and hyperbolic functions, and be able to apply these functions to real-life situations. In the inflation problem of Question 26(b), students needed to know that with the exponential function $P = A \times (1.2)^t$, the value of A was the initial price of the surfboard, which could be read from the graph. Many didn't.

The examiners were however pleased to note that a large number of students were able to solve the inverse variation problem in Question 28(b). Make sure you revise how to simplify algebraic expressions, solve equations (especially those involving fractions, powers and roots) and change the subject of a formula. Students are reminded that a formulas sheet is provided with your HSC exam.

6. Know your data analysis (statistics)

Data Analysis topics took up 23 marks of last year's HSC exam:

Preliminary course		
DA2: Data collection and sampling	2 marks	Question 25 (b) (iv): random sampling
DA3: Displaying single data sets	2 marks	Question 25 (b) (i), (ii): cumulative frequency and median
DA4: Summary statistics	1 mark	Question 8: mean
HSC course		
DA5: Interpreting sets of data	11 marks	Questions 15, 16, 21, 23(c), 25(b)(iii): comparing two data sets
DA6: The normal distribution	4 marks	Questions 19, 23(b): z-scores
DA7: Correlation	3 marks	Question 23(a): scatterplot and correlation

so this is another area that requires significant study and revision. General Mathematics students need to be familiar with the many different types of statistical displays, the statistical jargon and terminology, and be prepared to compare the data and displays of different distributions, commenting upon their shape, skewness, clustering and measures of location and spread. With the normal distribution, you must know the meanings of z-scores and the 68%, 95% and 99.7% limits.

Six more tips for Mathematics (2 unit and Extensions) students

1. Know your basic algebra and arithmetic

This tip is included here because of all of the Preliminary course topics, Basic Arithmetic and Algebra features most prominently in the HSC exam, mostly in Question 1. Last year, it took up 12 marks or 10% of the paper. In Question 1, students had to evaluate $\sqrt{\frac{3^2+12^2}{231-12^2}}$, solve an absolute value inequality and quadratic equation, simplify an expression involving algebraic fractions and calculate the original price of a VCR before 10% tax was added. As mentioned previously, examiners were surprised by the number of careless errors made by HSC students in Years 9-10 algebra skills such as factorising and expanding expressions, solving equations and inequalities, and substituting into formulas.

2. Learn your formulas

Each year, examiners report of students misquoting formulas or using one incorrectly, especially ones from coordinate geometry, trigonometry or series. In last year's Mathematics exam, they were surprised that many students did not remember the formula for the area of a parallelogram (which is learned in Year 8) for Question 2 (b) (iv), or the formula for the perpendicular distance between a point and a line for Question 2 (b) (iii).

Students are reminded that with financial applications of series, there are *no* loan repayments and superannuation formulas to be learned or memorised. Although they can be found in some misleading study guides, they are not part of the course and are not to be used. As every financial problem is different, any such formula used in an exam would need to be proved first. Students who tried to memorise and use a superannuation formula in Question 10(a) of last year's Mathematics paper experienced much difficulty and were generally unsuccessful.

3. Use the simplest method of solution

Every question in a mathematics exam has been designed to be answered within a specific timeframe. Use the marks allocated as a guide to how long it should take to complete a question and how much working is needed. Often, HSC students are guilty of using a more complicated and time-consuming method or approach when a simpler one would do the job more quickly. If your working-out to a problem is taking too long, then it is probably wrong or inappropriate (except perhaps in the case of a Mathematics Extension 2 question). A lot of wasted time and effort could be eliminated during an exam if you first spent a moment considering the best method to use. Could you draw a diagram or graph, use logic, make a list or table, guess-and-check or use your answers from the previous parts of the question? Not all problems need to be solved using fancy algebra.

Question 1(b) of the Mathematics paper was the absolute value inequality $| x + 3 | < 2$. There are numerous ways of solving such inequalities, some more complicated than others. The examiners found that students who rewrote this inequality as $-2 < x + 3 < 2$ achieved the most success.

The final part, (v), of the coordinate geometry Question 2(b) was "find the perpendicular distance from O to the line BC." This was best solved by working backwards from the answer to part (iv), the area of parallelogram OABC—by dividing it by the length of OA. However, most students missed this opportunity and "reinvented the wheel" instead, first finding the equation of the line BC and then using the perpendicular distance formula again, all for a simple 2-mark question.

Question 1(c) of the Mathematics Extension 1 paper was the sum $\sum_{n=4}^{7}(2n+3)$. This was easily evaluated by substituting into the formula to get 11 + 13 + 15 + 17 = 56, yet there were many who tried to apply series formulas for this one-mark question. Most of these students still got the question right, but there was much more margin for error with that method. Some students even thought too much about the question and used ${}^{n}C_{r}$, believing it was a binomial theorem problem.

Question 1 (e) of the same paper asked whether $x + 3$ was a factor of $x^3 - 5x + 12$. Extension 1 students who knew their factor theorem simply substituted -3 into the expression to see whether they got zero, but others took the long way round and did the long division. Again, this second method was more complicated and had a higher chance of error.

4. Practise setting out a geometrical proof correctly

Many HSC students have difficulty in presenting a geometrical proof clearly and logically. Examiners often find "essay answers" going over a page for a 2-mark geometry question. Such long answers are either wrong or inappropriate, otherwise the question would be worth a lot more marks. To answer such questions correctly, you need to read the question, draw a big diagram, identify what needs to be found, sketch a rough proof on the diagram, then write each step and its reasoning (in brackets) formally, presenting your proof as succinctly as possible.

Question 4(b) of the Mathematics paper required a proof involving an isosceles triangle:

[geometrical diagram from Q4(b) goes here]

(i) Show that $\angle ABC = 180° - 2x°$.

To score the full 2 marks in (i), students needed to state that:

$\angle CML = \angle CLM = x°$ (equal angles in isosceles ΔCML),
$\therefore \angle ACB = 180° - 2x°$ (angle sum of ΔCML)
$\therefore \angle ABC = \angle ACB = 180° - 2x°$ (given).

Students who listed the above statements without reasons scored no marks, while students who started the proof with $\angle ACB = 180° - 2x°$ without justifying with *both* the isosceles triangle and angle sum properties lost a mark.

With deductive geometry, you need to give sufficient reasons for all statements and use correct terminology. Don't say "Z angles" when you mean alternate angles or "butterfly angles" when you mean angles in the same segment of a circle. Such slang terms may be useful learning aids but they are neither widely used nor acceptable terms. In the circle geometry Question 3(b) of the Mathematics Extension 1 paper and Question 6(b) of the Mathematics Extension 2 paper, examiners reported students using the phrases "windsurfer theorem," "tangent theory" and "angle in arc" to mean the alternate segment theorem (the angle between a tangent and a chord is equal to the angle in the alternate segment). Don't do this because no-one will know what you mean and you will lose marks!

5. Know your logarithms and exponentials

Although the Logarithmic and Exponential Functions topic features frequently in the HSC Mathematics exam, students have a fairly superficial understanding of its concepts, possibly because it is a relatively new topic to them and one of the last to be taught in the year. There were five questions on this topic in last year's paper, worth a total of 6 marks, mainly based on the logarithm laws. Common mistakes included not being able to simplify ln 5 – ln 4 in Question 3 (a) and rewriting $\ln(x^2 - 9)$ as $\ln x^2 - \ln 9$ and $(e^x)^2$ as e^{x^2} in Question 3 (c). Make sure that you understand the logarithm and exponential laws and that you are able to use them to simplify, evaluate or solve expressions and equations involving ln x and e^x. Know the derivatives of exponential and logarithmic functions and their associated integrals. Understand that ln x and $\log_e x$ mean the same thing, and when calculating natural logarithms use the ln key on your calculator, not the log key (which actually stands for $\log_{10} x$).

Question 8(a) of the Mathematics paper was an exponential growth problem which was generally well done. Students should know that in the exponential function $N = N_o e^{kt}$, N_o is the initial value or population while k is the growth constant. In these problems, k is usually a decimal so it is important that students do not round off its value otherwise the accuracy of the final answer will be affected. The unrounded value of k can be left on the calculator and recalled (from display, memory or the ANS key) when needed in further calculations.

6. Know your calculus, especially its applications to the physical world

You should know by now that most of the HSC Mathematics course is taken up by calculus topics, with the only non-calculus topics being series, probability and geometry. Calculus topics from the HSC course took up 53 marks (44%) of last year's exam,

Applications of calculus to the physical world	20 marks
Geometrical applications of differentiation (curve sketching, maxima/minima)	10 marks
Integration	9 marks
Trigonometric functions	8 marks
Logarithmic and exponential functions	6 marks

so any serious study program should include thorough revision of them. The Applications of Calculus to the Physical World topic includes motion in a straight line (Question 7(c)), exponential growth (Question 8(a)) and rates of change (Question 9(b)). For Extension 1 students, it includes projectile motion (Question 4(b)), simple harmonic motion (Question 4(c)), and acceleration as a function of x (Question 7(a)).

In Question 5(d) where the trapezoidal rule was used to approximate an area, some students had trouble applying the formula or accidentally used Simpson's rule. As in previous years, the most successful students applied the formula to each of the 3 strips and totalled the answers, while the least successful used a complicated formula involving $\frac{b-a}{n}$ instead of h and thus had difficulty identifying the width of a strip.

One area of calculus that is often tested in HSC exams but not promoted much in textbooks and study guides is the interpretation of the size and sign of the derivative $\frac{dy}{dx}$ and second derivative $\frac{d^2y}{dx^2}$, either on graphs or as rates of change. These are those strange, unconventional high-level questions where you are given a graph of y = f(x) or y = f'(x) and you are asked to describe what's happening, draw another graph, or relate it to a real situation. Question 8(c) of last year's Mathematics Paper asked students to examine the rate of change $\frac{dy}{dt}$ of the water level in a curved bottle (2 marks) and then graph the function y = f(t) of the water level over time (another 2 marks). Over 70% of students either did not attempt this difficult question or scored zero marks. To achieve success here, you really need to develop a deep understanding of the meaning of derivative, concavity and points of inflexion, and then gain plenty of practice in answering these types of questions (check past HSC papers).

Students are also reminded to use the table of standard integrals provided at the back of every exam paper because every year there are a few who miss out on valuable marks by neglecting to refer to this page.

10 STEPS FOR SUCCESS IN HSC MATHEMATICS EXAMS

1. Find out all you can about the format of the exam (review past HSC papers)
2. Be prepared
3. Use the reading time and marking scheme to plan your approach to the exam. Note the harder questions—you may need to spend more time on them.
4. Spend the first minute of each question planning, thinking and choosing the best strategy to use. You don't need to be writing all of the time. What you're writing may be wrong and a waste of time.
5. Spread out your work. Draw big diagrams.
6. Pace yourself—keep an eye on the time.
7. Make sure you have answered the question. Check that it sounds reasonable, especially if it involves money or measurement. Did you include the correct units?
8. Don't get bogged down—move on if you're getting nowhere. If your working-out to a hard question is taking too long, then it's probably *wrong*. Retrace your steps, start again, or come back to it later.
9. If you make a mistake, draw a neat line through it. Don't scribble over it or use correction fluid. You may still get marks for it if it is right.
10. At the end of the exam, check your work and go back and attempt harder or uncertain questions.

References

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